# Automata and Formal Languages <br> Lecture 02 

## Books



## PowerPoint

http://www.bu.edu.eg/staff/ahmedaboalatah14-courses/14767


## FINITE AUTOMATA Regular Language

## Agenda

>Regular Languages
>Example: Regular Languages
>Finite Automata
>Example Finite Automaton
>Finite Automata (FA) as Accepter
>Transition Function \& Transition Table
$>$ Nondeterministic Finite Automata (NFA)
> Example: DFA
PExamples Find DFA \& NFA
PExample: What is the language?

## Regular Languages

The collection of regular languages over $\boldsymbol{A}$ is defined inductively as follows:

- Basis: $\boldsymbol{\Phi},\{\varepsilon\}$, and $\{\mathrm{a}\}$ are regular languages for all a $\in A$.
- Induction: If $\boldsymbol{L}$ and $\boldsymbol{M}$ are regular languages, then the following languages are also regular: $\mathbf{L \cup M}, \mathbf{M} . L$, and $L^{*}$.


## Example : Regular Languages

Regular languages over the alphabet $A=(a, b)$ :
$\Phi, \quad\{\varepsilon\}, \quad\{a\}, \quad\{b\}$
$\{\varepsilon, b\}=\{\varepsilon\} \cup\{b\}$
$\{a, a b\}=\{a\} .\{\varepsilon, b\}$
$\left\{\varepsilon, b, b b, \ldots, b^{n}, \ldots\right\}=\{b\}^{*}$
$\left\{a, a b, a b b, \ldots, a b^{n}, ..\right\}=\{a\}\left(\varepsilon, b, b b, \ldots, b^{n}, \ldots\right\}=\{a\} .\{b\}^{*}$
$\left\{\varepsilon, a, b, a a, b b, \ldots, a^{n}, b^{n}, \ldots\right\}=\{a\}^{*} \cup\{b\}^{*}$

## Finite Automata

A finite Automata or FA is defined by the

$$
M=\left(Q, \Sigma, \delta, q_{0}, F\right)
$$

where

- Q is a finite set of states,
- $\Sigma$ is a finite set of symbols called the input alphabet,
$\circ \delta: Q \times \Sigma \rightarrow Q$ is a total function called the transition function,
${ }^{\circ} q_{0} \in Q$ is the initial state,
${ }^{\circ} \mathrm{F} \subseteq \mathrm{Q}$ is a set of final states.


## Example Finite Automaton

A path $0,1,1,3$ with edges labeled $a, b$, a. Since 0 is the start state and 3 is a final state, we conclude that the FA accepts the string aba.

The FA also accepts the string baaabab by traveling along the path $0,2,2,2,2,3,3,3$.


# Finite Automata (FA) as Accepter 

$>$ A FA accepts a string $\boldsymbol{w}$ in $\boldsymbol{A}^{*}$ if there is:

- a path from the start state to some final state such that $\boldsymbol{w}$ is the concatenation of the labels on the edges of the path.
- Otherwise, the FA rejects w.
$>$ The set of all strings accepted by a finite Automata $\boldsymbol{M}$ is called the language of $\boldsymbol{M}$ and is denoted by $L(M)$.


## Transition Function \& Transition Table

$\delta\left(q_{0}, a\right)=q_{0} \quad \delta\left(q_{1}, b\right)=q_{2}$
$\delta\left(\mathrm{q}_{0}, \mathrm{~b}\right)=\mathrm{q}_{1} \quad \delta\left(\mathrm{q}_{2}, \mathrm{a}\right)=\mathrm{q}_{2}$
$\delta\left(q_{1}, a\right)=q_{2} \quad \delta\left(q_{2}, b\right)=q_{2}$

|  | $a$ | $b$ |
| :--- | :--- | :--- |
| $q_{0}$ | $q_{0}$ | $q_{1}$ |
| $q_{1}$ | $q_{2}$ | $q_{2}$ |
| $q_{2}$ | $q_{2}$ | $q_{2}$ |




## Nondeterministic Finite

 Automata (NFA)1. Edge with $\boldsymbol{\varepsilon}$.
2. Missing labels
3. Multiply edges start from the same state with
 the same label

## Example: DFA



The set $(a, b)^{*}$ of all strings over $\{a, b\}$


The set of all strings over $\{a, b\}$ that begin with $a$

## Examples Find DFA \& NFA

Regular languages over the alphabet $\mathrm{A}=(\mathrm{a}, \mathrm{b})$ :
$\Phi,\{\varepsilon\},\{a\},\{b\}$
$\{\varepsilon, b\}=\{\varepsilon\} \cup\{b\}$
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$\left\{\varepsilon, b, b b, \ldots, b^{n}, \ldots\right\}=\{b\}^{*}$
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## Example: What is the language?




